



## **Background**

This policy is a key update following the phased introduction of the 'bar model' method to help pupils visualise problems. It contains the key pencil and paper procedures that will be taught within our school. It has been written to ensure consistency and progression throughout the school and reflects a whole school agreement.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of mathematics. The mental methods for teaching mathematics will be taught systematically from Reception onwards and pupils will be given regular opportunities to develop the necessary skills. However, mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it.

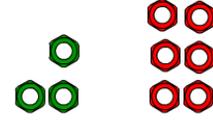
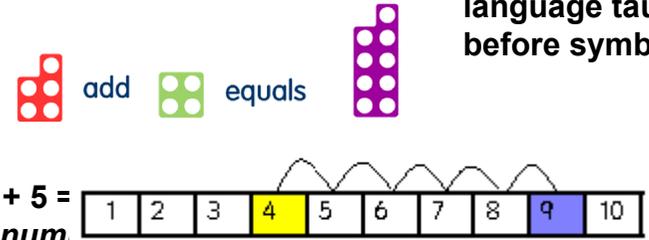
In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and to develop new ideas. Written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at Forest children will be encouraged to see mathematics as both a written and spoken language. Teachers will support and guide children through the following important stages:

- Using concrete apparatus to model and represent;
- Using pictures, models, images, and a mixture of words and symbols to represent numerical activities;
- Using standard symbols and conventions;
- Using jottings to aid a mental strategy;
- Using pencil and paper procedures;
- Using a calculator.

It is important that children do not abandon concrete apparatus, the using of images to represent, and then jottings and mental methods once pencil and paper procedures are introduced. Children will always be encouraged to look at a calculation/problem and then decide the best method to choose – pictures, mental calculation with or without jottings, structured recording or a calculator. The long-term aim is for all children to be able to select an efficient method of their choice (whether this be mental, written or using a calculator) that is appropriate for a given task. They will do this by always asking themselves:

- 'Can I do this in my head?'
- 'Can I do this in my head using drawings or jottings?'
- 'Do I need to use a pencil and paper procedure?'
- 'Do I need a calculator?'

Addition	
P Level 5	<p>Awareness of contrasting quantities e.g. 1 apple compared to lots of apples.</p> 
P Level 6 (EYFS PSRN) C1, C2	<p>Demonstrate an understanding of the concept of n e.g. 6 counters versus 3 counters</p> 
P Level 7 (EYFS PSRN) C1, C2, C3	<p>In practical situations, respond to adding 1 to a group of ob e.g. add 1 pencil to pencils in pot</p> 
P Level 8 (EYFS PSRN) C4, C7	<p>In practical situations, respond to adding 1 to a group of objects and identifying the ne e.g. add 1 pencil to pencils in pot and say how many now "1 more than 3 is 4"</p> 
NC Level 1 (EYFS PSRN) C9	<p>Use addition to find the total of two or more sets of objects. Understands when to use symbols such as +, = Recognise that addition can be done in any order. language taught before symbols</p> <p> <math>3 + 4 = \bullet</math>   <math>3 + \bullet = 7</math>   <math>\bullet + 4 = 7</math>   <math>\bullet + \nabla = 7</math>  <math>\bullet = 3 + 4</math>   <math>7 = 3 + \bullet</math>   <math>7 = \bullet + 4</math>   <math>7 = \bullet + \nabla</math> </p>  <p> <b>Addition by stepping along a number track</b>  <i>Initially in given order progressing to counting on from largest num.</i> </p> <p> <b>Addition by counting on along a numbered number line</b>  <i>Initially in given order progressing to counting on from largest</i> </p>

Addition

## Stage 1

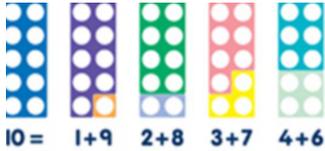
### + = signs and missing numbers

$$3 + 4 = \bullet \quad \bullet = 3 + 4$$

$$3 + \bullet = 7 \quad 7 = \bullet + 4$$

$$\bullet + 4 = 7 \quad 7 = 3 + \bullet$$

$$\bullet + \nabla = 7 \quad 7 = \bullet + \nabla$$



3 + 4 is the same as 7 as modelled using Numicon

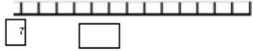
Use Numicon to further understand the equivalence in a number sentence.

Promoting covering up of operations and numbers.

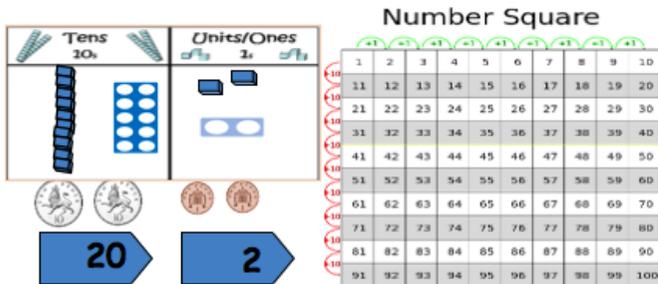
### Using Number lines

(Teacher model number tracks and lines with numbers and with missing numbers)

7 + 4 = 11 Children go up in 1s

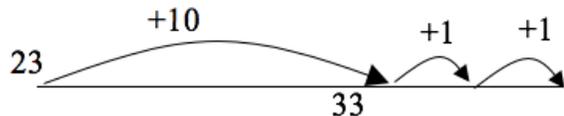


Able to use a hundred square securely for addition.



### Partition the second number only

$$23 + 12 = 23 + 10 + 1 + 1 = 33 +$$



$$\square + \square = \square \rightarrow \square = \square + \square$$

## Stage 2

### + = signs and missing numbers

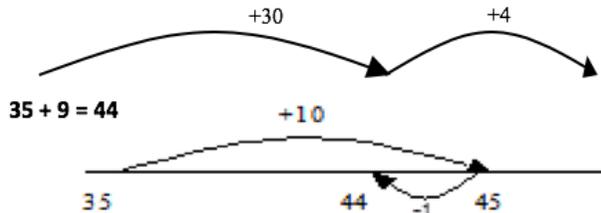
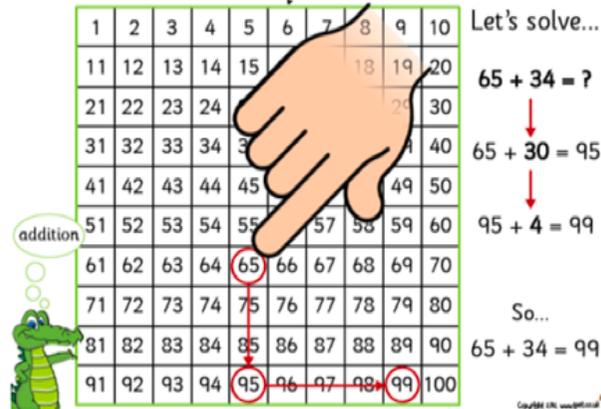
#### Adding three numbers

$$34 + 5 = 30 + \square + \square$$

$$32 + \square + \square = 100 \quad 35 = 1 + \square + 5$$

#### Partition into tens and ones

How to use a hundred square...



### Secure with addition

$$123 = 100 + 20 + 3$$

$$+ 45 = \quad \quad 40 + 5$$

$$168 = 100 + 60 + 8$$

HTU

$$\begin{array}{r} 123 \\ + 45 \\ \hline 168 \end{array}$$

Is the same as:  $\begin{array}{r} 123 \\ + 45 \\ \hline 168 \end{array}$

$$\square = \square + \square \rightarrow \square + \square = \square + \square$$

## Stage 3

### Partition into tens and ones and recombine

#### Consolidate:

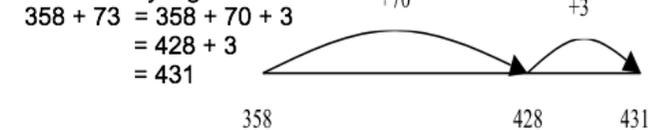
$$36 + 53 = 53 + 30 + 6$$

$$= 83 + 6$$

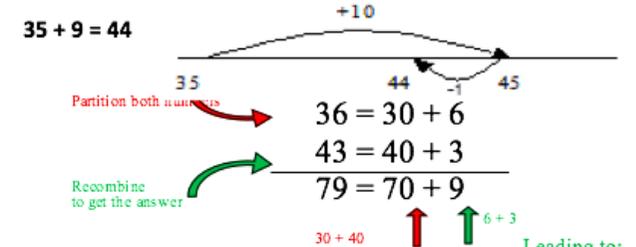
$$= 89$$

### Partition into hundreds, tens and ones and recombine

Partition the second number only e.g.



### Adding 9 or 11 by adding 10 and adjusting by 1



$$149 = 100 + 40 + 9$$

$$35 = \quad \quad 30 + 5$$

$$184 = 100 + 70 + 14$$

### Able to use columnar addition

Remember to line up the HTU.

$$\begin{array}{r} 467 \\ + 215 \\ \hline 682 \end{array}$$

Because 7 + 5 = 12 we have to carry the 10.



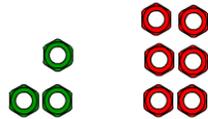
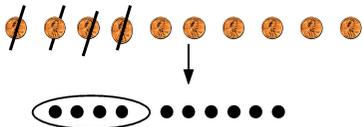
$$\square + \square = \square + \square \rightarrow$$

$$\square + \square = \square + \square \rightarrow \square + \square = \square - \square$$





## Subtraction

P Level 5	Awareness of contrasting quantities eg 1 apple compared to lots of apples.	
P Level 6	<i>No new subtraction calculation objectives</i>	
P Level 7 (EYFS PSRN) C1, C2	Demonstrate an understanding of the concept of eg 3 counters versus 6 counters	
P Level 8 (EYFS PSRN) C3,C5,C7	In practical situations, respond to taking 1 from a group of objects and identifying the new amount. eg take away 1 pencil from the pencils in pot and say how many now "1 less than 4 is 3"	
NC Level 1 (EYFS PSRN) C9	<p>Understand subtraction as "take away" Recording using pictures or marks</p> <p>Sam spent 4p. What was his change from 10p?</p> <p style="text-align: center;">  </p> <p>Understands when to use symbols such as -, = language taught before symbols</p> <p>  take away is   minus equals         </p> <p>  the difference between and is         </p> <p><b>- , = signs and missing numbers</b></p> <p>7 - 4 = •    7 - • = 3    • - 4 = 3    • - 3 = 4</p>	<p>Understand subtraction as "find the difference" "How many more do we need to add on to make?"</p> <p style="text-align: center;">  </p> <p>"What do you have to add to 4 to get 6?" or "What is the difference between 4 and 6?"</p> <p style="text-align: center;">"What is the difference between 2 and 9?"</p> <p style="text-align: center;"><math>9 - 2 = 7</math></p>

# Subtraction

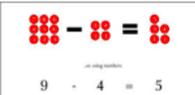
## Stages 1&2

### Pictures / marks

Sam spent 4p. What was his change from 10p



### Visual / practical activities



= signs at missing

### numbers

$$7 - 3 = \square$$

$$7 - \square = 4$$

$$\square - 3 = 4$$

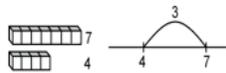
$$\square - \square = 4$$

$$\square = 7 - 3$$

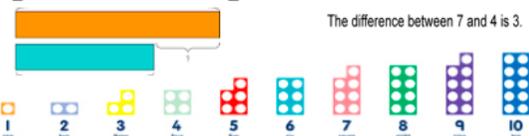
$$4 = \square - 3$$

$$4 = 7 - \square$$

$$4 = \square - \square$$

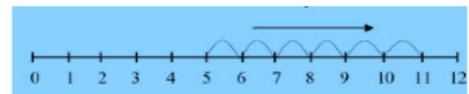


The difference between 7 and 4 is 3.



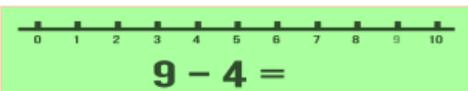
### Number lines

Understand subtraction as finding the "difference"



Establish **counting on** as a strategy when the numbers are

Record by - drawing jumps on prepared lines



### Understand subtraction as take-away

$$10 - 3 = \underline{\quad}$$

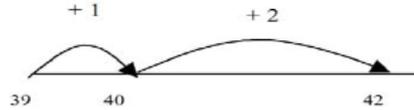


Establish **counting back** as a strategy.

## Stages 3&4

### - = signs and missing numbers

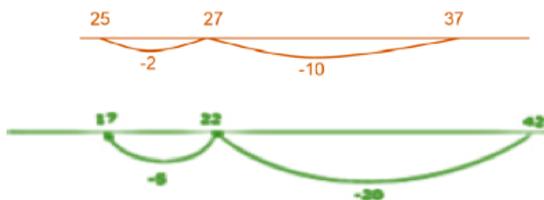
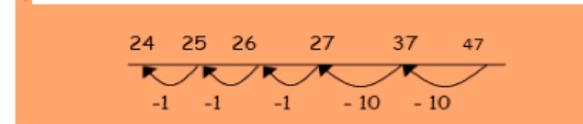
Continue using a range of equations as in Stage 1 but with appropriate numbers.



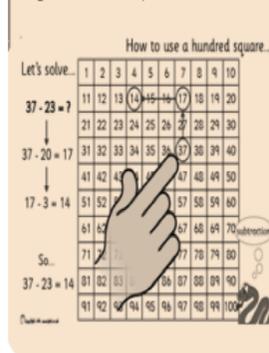
Consolidate counting on as a strategy where there is a small difference between the numbers

$$37 - 12 =$$

Leading to **counting back**, first in 10s then 1s.



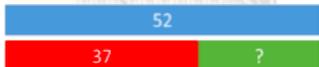
Using a Hundred Square:



Partition both numbers.  
 $67 - 32 = 35$   
 $67 = 60 + 7$   
 $32 = 30 + 2$   
 $35 = 30 + 5$   
 $60 - 30 = 30$   
 $7 - 2 = 5$   
 lines up the tens and units.  
 Recombine to get the answer.

$$181 - 57$$

$$\begin{array}{r} 181 \\ - 57 \\ \hline 124 \end{array}$$



$$52 - ? = 37$$

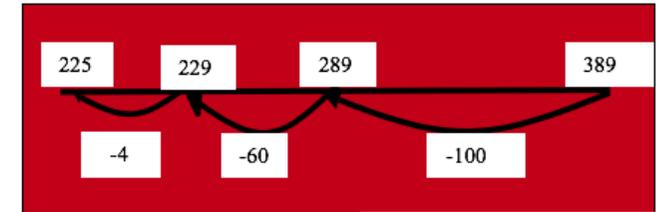
## Stages 5&6

Find a small difference by counting on

$$102 - 97 = \quad 508 - 317 = \quad 1002 - 781 =$$

**Note:** Counting back is not always the most efficient method when the numbers are closer together. Reinforce concept with practical strategies essential to see 'difference'.

$$389 - 164 = 225$$



Leading to expanded method without borrowing

$$774 - 432 =$$

$$774 = 700 + 70 + 4$$

$$432 = 400 + 30 + 2$$

Leading to expanded method with borrowing

$$935 - 587 =$$

$$935 = 800 + 100 + 30 + 5$$

$$587 = 500 + 80 + 7$$

$$342 = 300 + 40 + 2$$

$$348 = 300 + 40 + 8$$

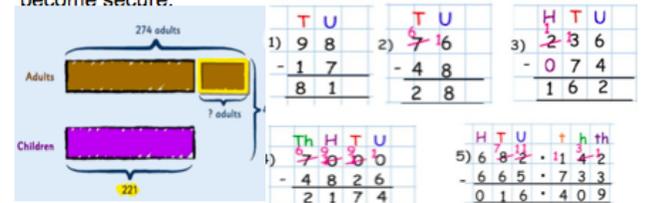
$$889 - 647 = 442$$

$$1004 - 692 = 442$$

$$\begin{array}{r} 889 \\ - 647 \\ \hline 442 \end{array}$$

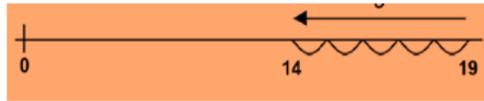
$$\begin{array}{r} 1004 \\ - 692 \\ \hline 442 \end{array}$$

Use both expanded and compact method together until pupils become secure.



$$\square - \square = \square \quad \square + \square = \square - \square$$

Constructing own lines, if appropriate: -5



Leading to **counting back**, first in 10s then 1s

## Multiplication

P Level 5

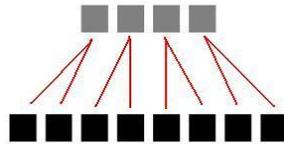
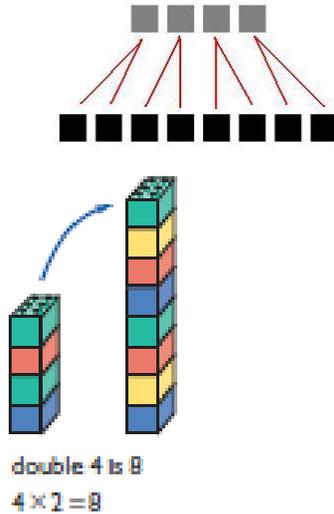
P Level 6

P Level 7

P Level 8

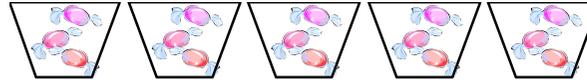
NC Level  
1

Recall doubles of numbers up to 10  
Doubles ( $4 + 4 = 8$ )



Solve practical problems that involve combining groups of 2, 5 or 10,

There are 3 sweets in one bag.  
How many sweets are there in 5 bags?



## Multiplication

## Stage 1

### Pictures and symbols

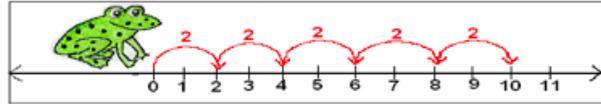
Counting in 2's e.g. socks, shoes, Counting in 5's  
Counting in 10's e.g. fingers, toes...



### Counting in 2p's, 5p's, 10p's



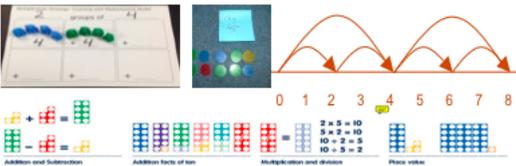
(Recording on a number line modelled by the teacher when solving problems)



Use bead strings, bars & Numicons to model groups of



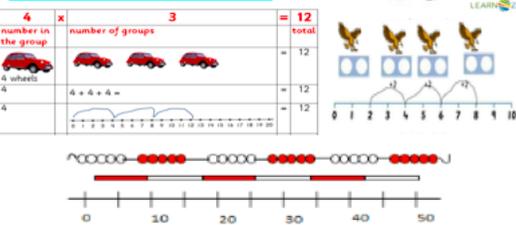
Use cubes and pegs to show arrays and repeated addition leading on to number line.



Begin to learn 2, 5 and 10 times tables.

### x = signs and missing numbers

$7 \times 2 = \square$      $\square = 2 \times 7$   
 $7 \times \square = 14$      $14 = \square \times 7$   
 $\square \times 2 = 14$      $14 = 2 \times \square$   
 $\square \times \nabla = 14$      $14 = \square \times \nabla$



## Stage 2

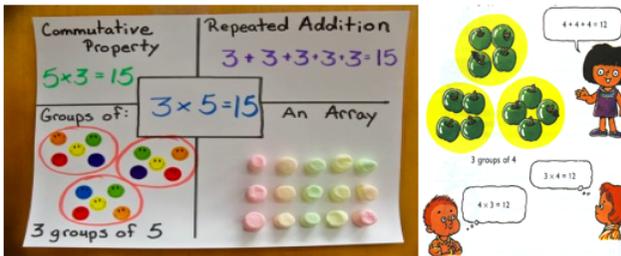
### x = signs and missing numbers

$7 \times 2 = \square$      $\square = 2 \times 7$   
 $7 \times \square = 14$      $14 = \square \times 7$   
 $\square \times 2 = 14$      $14 = 2 \times \square$   
 $\square \times \nabla = 14$      $14 = \square \times \nabla$

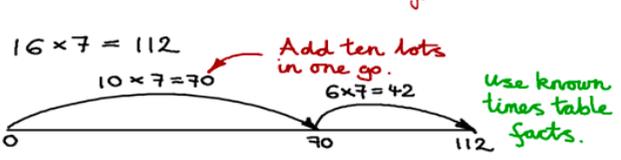
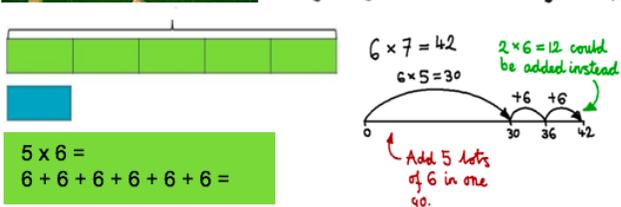
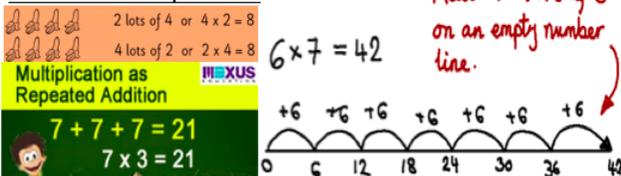


### Arrays and repeated addition

• • • •     $4 \times 2$  or  $4 + 4$   
 • • • •     $2 \times 4$   
 or repeated addition  $2 + 2 + 2 + 2$



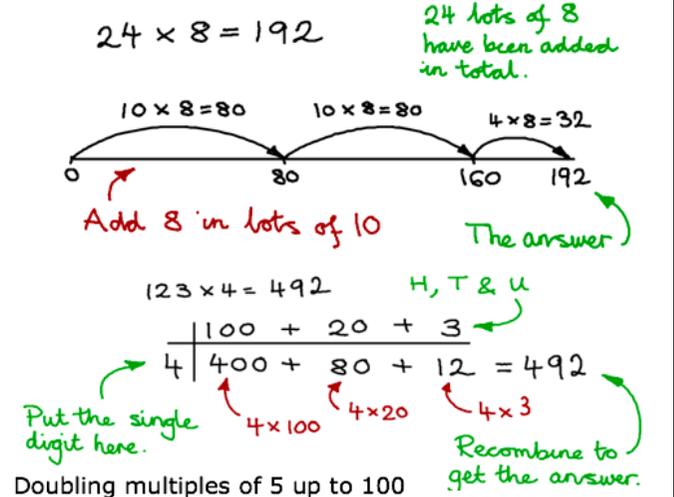
Repeated addition using a number line. Understanding multiplication as repeated addition is key to understanding formal methods of multiplication



## Stage 3

### x = signs and missing numbers

Continue using a range of equations as in Stage 2 but with appropriate numbers.



Doubling multiples of 5 up to 100

$15 \times 2 = 30$

Partition  $(10 \times 2) + (5 \times 2)$

$20 + 10 = 30$

	H	T	U	Fact Box
1) $\begin{array}{r} 35 \\ \times 5 \\ \hline 175 \end{array}$				
2) $\begin{array}{r} 36 \\ \times 8 \\ \hline 288 \end{array}$				$2 \times 8 = 16$
				$3 \times 8 = 24$
				$5 \times 8 = 40$
				$6 \times 8 = 48$

### Missing Numbers

Solve balancing problems showing a good understanding of the role of the '=' symbol. E.g.  $7 \times 10 = 82 - \Delta$

### Grid Method

X	30	5
7	210	35

$210 + 35 = 245$

Make Level 3 RPP calculations and multiplication of TU numbers by 2, 3, 4, 5 and 10.

# Multiplication

## Stage 4

### Pencil and paper procedures

Grid method

$123 \times 4 = 492$

Partition the number into H, T & U

100	+	20	+	3
4				
400	+	80	+	12

$400 + 80 + 12 = 492$

Put the single digit here.

$4 \times 100$     $4 \times 20$     $4 \times 3$

Recombine to get the answer.

Grid method

$56 \times 43 = 2408$

Partition both numbers.

50	+	6
x		
40	+	3
2000	+	240
150	+	18
+ 168		
2408		

Recombine the rows.

Multiply the top numbers by the side.

Add to get the total.

H	T	U	Fact Box
3	6		$2 \times 8 = 16$
	x	8	$3 \times 8 = 24$
2	8	8	$5 \times 8 = 40$
			$6 \times 8 = 48$

## Stage 5

### Grid Method & Grid Method (Decimals)

As in Stage 4 but with HTU x TU and ThHTU x HTU and similar appropriately sized numbers.

Encourage estimations and approximations.

### x = signs and missing numbers

Th H T U

Multiply the top number by the units of the bottom.

3	2	4
x	2	3
972		
6	4	80
7452		
11		

$(3 \times 4) + (3 \times 20) + (3 \times 300)$

Multiply the top number by the tens of the second number.

Add to get the answer.

Th	H	T	U
	5	3	
x		2	6
318 ← Answer line 1			
1060 ← Answer line 2			
1378 ← Answer line 3			

123
x 45
615 ( $123 \times 5$ )
4920 ( $123 \times 40$ )
5535 ( $615 + 4920$ )
1

6	2	8	
x	2	3	7
4396 ones			
18840 tens			
125600 hundreds			
148836			

Multiplying decimal numbers using the grid method.

	5	+	0.2
6	30	+	1.2
= 31.20			
0.3	1.5	+	0.06
= 1.56			
32.76			

$0.3 \times 0.2$

Take care to line up the digits. Adding a place holder will help.

## Stage 6

### x = signs and missing numbers

Pencil and paper procedures

$3.77 \times 2.8$

**3.77 (2 decimal places)**

**2.80 (2 decimal places)**

Remove the decimals and multiply, then add the decimal point after counting the decimal places in the question.

377
x 280
30160
75400
10.5560

2	7	8	
0	2	2	3
1	0	2	3
9	8	8	2
1	4	5	2

$278 \times 34 = 9,452$

3	1	7	x
1	5	5	3
2	4	0	5
8	3	8	6

$317 \times 58 = 18,386$

**$(6 - 9) \times 10 \div -3 = ?$**

$(6 - 9) \times 10 \div -3$

$= -3 \times 10 \div -3$

$= -30 \div -3$

$= +10$

$= 10$  ✓

**BODMAS** Pendas

**BODMAS** Pendas  
Do Division and Multiplication working from left to right.

B	O	D	M	A	S
or		or			
M					

Exponent (index or power)

Base

$6^3 = 6 \times 6 \times 6$

Shorthand way of representation

Normal representation (Base multiplied exponent number of times)

$\square + \square \times \square = 20$

$\square + \square + \square = 20$

$\square - \square \times \square = -20$

$= 20$     $= 20$     $= 10$

## Division

P Level 5

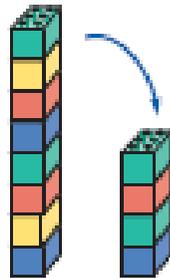
P Level 6

P Level 7

P Level 8

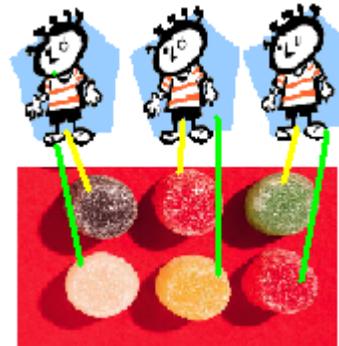
NC Level  
1

Recall halves of even numbers up to 10



half of 8 is 4

Share equally (practically)  
If I want to share six sweets fairly between 3 children,  
how many does each child get?

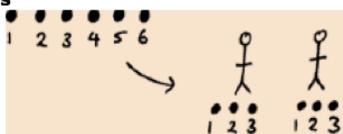


# Division

## Stage 1

### Pictures / marks

6 mince pies are shared equally between 2 people. How many does each one get?



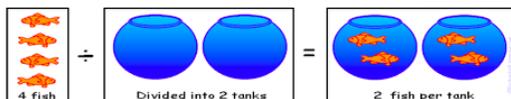
### Use practical resources – cubes & counters

$$6 \div 2 = \square \quad \square = 6 \div 2$$

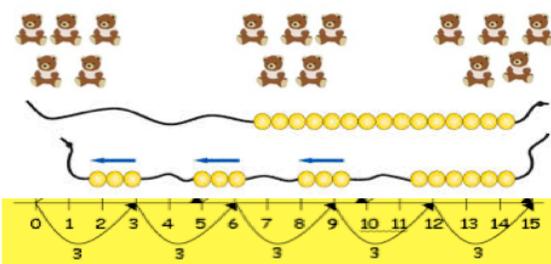
$$6 \div \square = 3 \quad 3 = 6 \div \square$$

$$\square \div 2 = 3 \quad 3 = \square \div 2$$

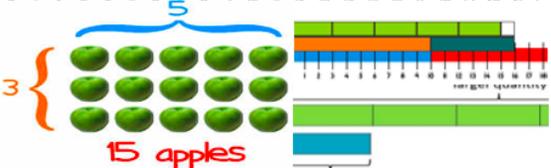
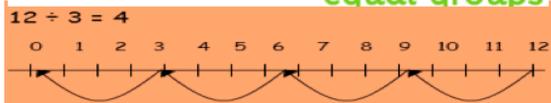
$$\square \div \nabla = 3 \quad 3 = \square \div \nabla$$



$$15 \div 3 = 5 \text{ 'shared between } 3 =$$



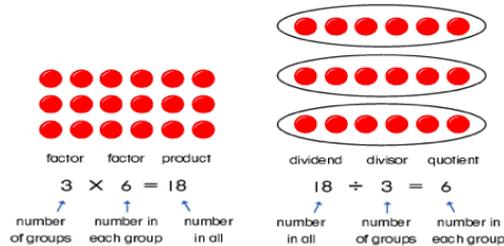
equal groups



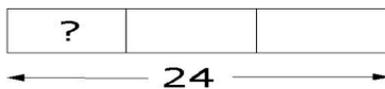
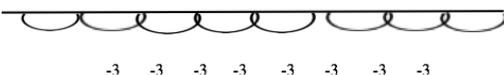
## Stage 2

### ÷ = signs and missing numbers

Understand division as sharing and grouping



$$24 \div 3 =$$



$$24 \div 3 = ?$$

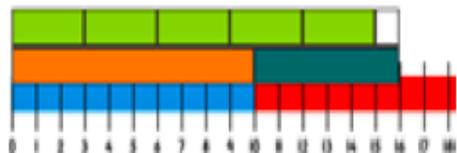
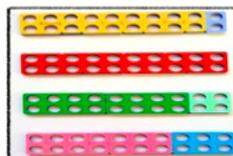


$$20 \div 3 = 6r2$$

$$20 \div 5 = 4$$

$$20 \div 8 = 2r4$$

$$20 \div 7 = 2r6$$



## Stage 3

### ÷ = signs and missing numbers

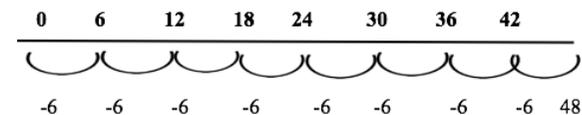
Continue using a range of equations as in Stage 2 but with appropriate numbers.

Understand division as sharing and grouping

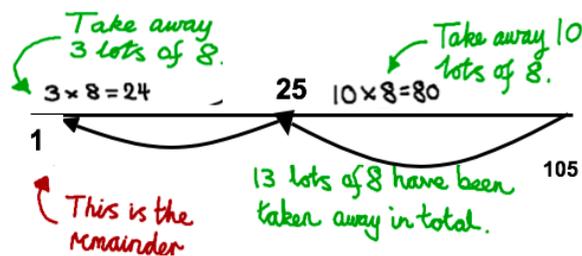
Use repeated subtraction.

Subtract 6 repeatedly

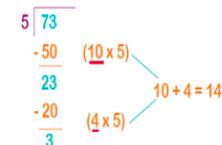
$$48 \div 6 =$$



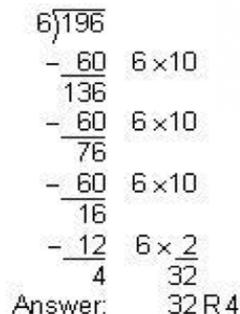
8 lots of 6 has been taken



$$73 \div 5$$



How many 5s have been subtracted?  
14 sets of 5, with 3 left over.  
Answer:  $73 \div 5 = 14 r3$



# Division

## Stage 4

÷ = signs and missing numbers

$$72 \div 9 = 8$$

*The dividend goes here.*

$$\begin{array}{r} 9 \overline{) 72} \\ -45 \quad (5 \times 9) \\ \hline 27 \\ -27 \quad (3 \times 9) \\ \hline 0 \end{array}$$

*The divisor goes here.*

*Take away 5 lots then 3 lots of 9.*

Next Steps:

Chunking with remainders.

$$76 \div 8 = 9 \text{ r } 4$$

$$\begin{array}{r} 8 \overline{) 76} \\ -72 \quad (9 \times 8) \\ \hline 4 \end{array}$$

*9 lots have been taken away.*

*This is the remainder.*

Chunking using times table facts.

Children will continue to explore division as repeated subtraction. They will use their increasing knowledge of times tables to subtract in larger chunks.

$$128 \div 7 = 18 \text{ r } 2$$

$$\begin{array}{r} 7 \overline{) 128} \\ -70 \quad (10 \times 7) \\ \hline 58 \\ -35 \quad (5 \times 7) \\ \hline 23 \\ -21 \quad (3 \times 7) \\ \hline 2 \end{array}$$

*Use the 10 times table to subtract lots of 7.*

*Subtract using known times table facts.*

*The remainder.*

Chunking is best used for 2 or more digit divisors, whilst short division is better for 1 digit or simple 2 digit divisors

$$\begin{array}{r} 115 \text{ r } 4 \\ 8 \overline{) 912} \end{array}$$

## Stage 5

Remainders

$$369 \div 14 = 26 \text{ r } 5$$

$$\begin{array}{r} 14 \overline{) 369} \\ -280 \quad (20 \times 14) \\ \hline 89 \\ -70 \quad (5 \times 14) \\ \hline 19 \\ -14 \quad (1 \times 14) \\ \hline 5 \end{array}$$

*Subtract in the largest chunk possible.*

*26 lots have been taken away in total.*

Quotients expressed as fractions or decimal fractions

$$676 \div 8 = 84.5$$

Expressing the remainder as a fraction.

$$50 \div 4 = 12 \text{ r } 2$$

*The remainder.*

$$= 12 \frac{2}{4}$$

*The divisor.*

*This can then be converted into a decimal.*

This leads to using short division using decimals

$$\begin{array}{r} 1.38 \\ 3 \overline{) 4.14} \end{array}$$

$$\begin{array}{r} 137 \text{ r } 5 \\ 7 \overline{) 964} \end{array}$$

$$\begin{array}{r} 0.45 \\ 9 \overline{) 4.05} \end{array}$$

$$\begin{array}{r} 01.375 \\ 8 \overline{) 11.000} \end{array}$$

Long division

$$\begin{array}{r} 12 \text{ r } 6 \\ 24 \overline{) 294} \\ \underline{24} \phantom{0} \\ 45 \phantom{0} \\ \underline{48} \\ 6 \end{array}$$

## Stage 6

÷ = signs and missing numbers

$$\frac{4}{5} = \frac{0.8}{1} = 0.80 = 80\%$$

$$\begin{array}{r} 4 \phantom{0} \\ 5 \overline{) 4.0} \\ \underline{-4.0} \\ 0 \end{array}$$

$$8 \overline{) 48.3}$$

$$2.4 \overline{) 38.4}$$

$$48 \div 8 = 6$$

$$4.8 \div 0.8 = 6$$

$$480 \div 80 = 6$$

$$0.48 \div 0.08 = 6$$

$$30 \div 5 + 4 \times 2 + 14 = ?$$

$$30 \div 5 + 4 \times 2 + 14 \quad \text{BODMAS}$$

$$= 6 + 4 \times 2 + 14 \quad \text{BODMAS}$$

$$= 6 + 8 + 14 \quad \text{BODMAS}$$

$$= 6 + 8 + 14$$

$$= 14 + 14$$

$$= 28$$

Do Addition and Subtraction working from left to right.

1.	2.	3.	4.
B	O	D	A
or	or	or	or
M	S		

Ginny           

Paul           

$$7 + 63 \div 9 = \quad = ?$$

$\begin{array}{r} 17r7 \\ 14 \overline{)24^{105}} \end{array}$		
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# Teaching Children to Calculate Mentally

## Addition and Subtraction

<b>Recall:</b> <b>Children should be able to derive and recall:</b>	<b>Mental calculation skills:</b> <b>Working mentally, with jottings if needed, children should be able to:</b>	<b>Mental methods or strategies:</b> <b>Children should understand when to and be able to apply these strategies:</b>
<p><b>Year 1</b></p> <ul style="list-style-type: none"> <li>● number pairs with a total of 10, e.g. <math>3 + 7</math>, or what to add to a single-digit number to make 10, e.g. <math>3 + \square = 10</math></li> <li>● addition facts for totals to at least 5, e.g. <math>2 + 3</math>, <math>4 + 3</math></li> <li>● addition doubles for all numbers to at least 10, e.g. <math>8 + 8</math></li> </ul>	<ul style="list-style-type: none"> <li>● add or subtract a pair of single-digit numbers, e.g. <math>4 + 5</math>, <math>8 - 3</math></li> <li>● add or subtract a single-digit number to or from a teens number, e.g. <math>13 + 5</math>, <math>17 - 3</math></li> <li>● add or subtract a single-digit to or from 10, and add a multiple of 10 to a single-digit number, e.g. <math>10 + 7</math>, <math>7 + 30</math></li> <li>● add near doubles, e.g. <math>6 + 7</math></li> </ul>	<ul style="list-style-type: none"> <li>● reorder numbers when adding, e.g. put the larger number first</li> <li>● count on or back in ones, twos or tens</li> <li>● partition small numbers, e.g. <math>8 + 3 = 8 + 2 + 1</math></li> <li>● partition and combine tens and ones</li> <li>● partition: double and adjust, e.g. <math>5 + 6 = 5 + 5 + 1</math></li> </ul>
<p><b>Year 2</b></p> <ul style="list-style-type: none"> <li>● addition and subtraction facts for all numbers up to at least 10, e.g. <math>3 + 4</math>, <math>8 - 5</math></li> <li>● number pairs with totals to 20</li> <li>● all pairs of multiples of 10 with totals up to 100, e.g. <math>30 + 70</math>, or <math>60 + \square = 100</math></li> <li>● what must be added to any two-digit number to make the next multiple of 10, e.g. <math>52 + \square = 60</math></li> <li>● addition doubles for all numbers to 20, e.g. <math>17 + 17</math> and multiples of 10 to 50, e.g. <math>40 + 40</math></li> </ul>	<ul style="list-style-type: none"> <li>● add or subtract a pair of single-digit numbers, including crossing 10, e.g. <math>5 + 8</math>, <math>12 - 7</math></li> <li>● add any single-digit number to or from a multiple of 10, e.g. <math>60 + 5</math></li> <li>● subtract any single-digit number from a multiple of 10, e.g. <math>80 - 7</math></li> <li>● add or subtract a single-digit number to or from a two-digit number, including crossing the tens boundary, e.g. <math>23 + 5</math>, <math>57 - 3</math>, then <math>28 + 5</math>, <math>52 - 7</math></li> <li>● add or subtract a multiple of 10 to or from any two-digit number, e.g. <math>27 + 60</math>, <math>72 - 50</math></li> <li>● add 9, 19, 29, ... or 11, 21, 31, ...</li> <li>● add near doubles, e.g. <math>13 + 14</math>, <math>39 + 40</math></li> </ul>	<ul style="list-style-type: none"> <li>● reorder numbers when adding</li> <li>● partition: bridge through 10 and multiples of 10 when adding and subtracting</li> <li>● partition and combine multiples of tens and ones</li> <li>● use knowledge of pairs making 10</li> <li>● partition: count on in tens and ones to find the total</li> <li>● partition: count on or back in tens and ones to find the difference</li> <li>● partition: add a multiple of 10 and adjust by 1</li> <li>● partition: double and adjust</li> </ul>

<p><b>Year 3</b></p> <ul style="list-style-type: none"> <li>● addition and subtraction facts for all numbers to 20, e.g. <math>9 + 8</math>, <math>17 - 9</math>, drawing on knowledge of inverse operations</li> <li>● sums and differences of multiples of 10, e.g. <math>50 + 80</math>, <math>120 - 90</math></li> <li>● pairs of two-digit numbers with a total of 100, e.g. <math>32 + 68</math>, or <math>32 + \square = 100</math></li> <li>● addition doubles for multiples of 10 to 100, e.g. <math>90 + 90</math></li> </ul>	<ul style="list-style-type: none"> <li>● add and subtract groups of small numbers, e.g. <math>5 - 3 + 2</math></li> <li>● add or subtract a two-digit number to or from a multiple of 10, e.g. <math>50 + 38</math>, <math>90 - 27</math></li> <li>● add and subtract two-digit numbers e.g. <math>34 + 65</math>, <math>68 - 35</math></li> <li>● add near doubles, e.g. <math>18 + 16</math>, <math>60 + 70</math></li> </ul>	<ul style="list-style-type: none"> <li>● reorder numbers when adding</li> <li>● identify pairs totalling 10 or multiples of 10</li> <li>● partition: add tens and ones separately, then recombine</li> <li>● partition: count on in tens and ones to find the total</li> <li>● partition: count on or back in tens and ones to find the difference</li> <li>● partition: add or subtract 10 or 20 and adjust</li> <li>● partition: double and adjust</li> <li>● partition: count on or back in minutes and hours, bridging through 60 (analogue times)</li> </ul>
<p><b>Year 4</b></p> <ul style="list-style-type: none"> <li>● sums and differences of pairs of multiples of 10, 100 or 1000</li> <li>● addition doubles of numbers 1 to 100, e.g. <math>38 + 38</math>, and the corresponding halves</li> <li>● what must be added to any three-digit number to make the next multiple of 100, e.g. <math>521 + \square = 600</math></li> <li>● pairs of fractions that total 1</li> </ul>	<ul style="list-style-type: none"> <li>● add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. <math>47 + 58</math>, <math>91 - 35</math></li> <li>● add or subtract a near multiple of 10, e.g. <math>56 + 29</math>, <math>86 - 38</math></li> <li>● add near doubles of two-digit numbers, e.g. <math>38 + 37</math></li> <li>● add or subtract two-digit or three-digit multiples of 10, e.g. <math>120 - 40</math>, <math>140 + 150</math>, <math>370 - 180</math></li> </ul>	<ul style="list-style-type: none"> <li>● count on or back in hundreds, tens and ones</li> <li>● partition: add tens and ones separately, then recombine</li> <li>● partition: subtract tens and then ones, e.g. subtracting 27 by subtracting 20 then 7</li> <li>● subtract by counting up from the smaller to the larger number</li> <li>● partition: add or subtract a multiple of 10 and adjust, e.g. <math>56 + 29 = 56 + 30 - 1</math>, or <math>86 - 38 = 86 - 40 + 2</math></li> <li>● partition: double and adjust</li> <li>● use knowledge of place value and related calculations, e.g. work out <math>140 + 150 = 290</math> using <math>14 + 15 = 29</math></li> </ul>

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|  |  | <ul style="list-style-type: none"><li>● partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)</li></ul> |
|--|--|--|

<p><b>Year 5</b></p> <ul style="list-style-type: none"> <li>● sums and differences of decimals, e.g. <math>6.5 + 2.7</math>, <math>7.8 - 1.3</math></li> <li>● doubles and halves of decimals, e.g. half of 5.6, double 3.4</li> <li>● what must be added to any four-digit number to make the next multiple of 1000, e.g. <math>4087 + \square = 5000</math></li> <li>● what must be added to a decimal with units and tenths to make the next whole number, e.g. <math>7.2 + \square = 8</math></li> </ul>	<ul style="list-style-type: none"> <li>● add or subtract a pair of two-digit numbers or three-digit multiples of 10, e.g. <math>38 + 86</math>, <math>620 - 380</math>, <math>350 + 360</math></li> <li>● add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. <math>235 + 198</math></li> <li>● find the difference between near multiples of 100, e.g. <math>607 - 588</math>, or of 1000, e.g. <math>6070 - 4087</math></li> <li>● add or subtract any pairs of decimal fractions each with units and tenths, e.g. <math>5.7 + 2.5</math>, <math>6.3 - 4.8</math></li> </ul>	<ul style="list-style-type: none"> <li>● count on or back in hundreds, tens, ones and tenths</li> <li>● partition: add hundreds, tens or ones separately, then recombine</li> <li>● subtract by counting up from the smaller to the larger number</li> <li>● add or subtract a multiple of 10 or 100 and adjust</li> <li>● partition: double and adjust</li> <li>● use knowledge of place value and related calculations, e.g. <math>6.3 - 4.8</math> using <math>63 - 48</math></li> <li>● partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)</li> </ul>
<p><b>Year 6</b></p> <ul style="list-style-type: none"> <li>● addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place, e.g. <math>650 + \square = 930</math>, <math>\square - 1.4 = 2.5</math></li> <li>● what must be added to a decimal with units, tenths and hundredths to make the next whole number, e.g. <math>7.26 + \square = 8</math></li> </ul>	<ul style="list-style-type: none"> <li>● add or subtract pairs of decimals with units, tenths or hundredths, e.g. <math>0.7 + 3.38</math></li> <li>● find doubles of decimals each with units and tenths, e.g. <math>1.6 + 1.6</math></li> <li>● add near doubles of decimals, e.g. <math>2.5 + 2.6</math></li> <li>● add or subtract a decimal with units and tenths, that is nearly a whole number, e.g. <math>4.3 + 2.9</math>, <math>6.5 - 3.8</math></li> </ul>	<ul style="list-style-type: none"> <li>● count on or back in hundreds, tens, ones, tenths and hundredths</li> <li>● use knowledge of place value and related calculations, e.g. <math>680 + 430</math>, <math>6.8 + 4.3</math>, <math>0.68 + 0.43</math> can all be worked out using the related calculation <math>68 + 43</math></li> <li>● use knowledge of place value and of doubles of two-digit whole numbers</li> <li>● partition: double and adjust</li> <li>● partition: add or subtract a whole number and adjust, e.g. <math>4.3 + 2.9 = 4.3 + 3 - 0.1</math>, <math>6.5 - 3.8 = 6.5 - 4 + 0.2</math></li> </ul>

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|--|--|---|
|  |  | <ul style="list-style-type: none"><li>● partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times, 12-hour and 24-hour clock)</li></ul> |
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# Teaching Children to Calculate Mentally

## Multiplication and Division

<b>Recall:</b> <b>Children should be able to derive and recall:</b>	<b>Mental calculation skills:</b> <b>Working mentally, with jottings if needed, children should be able to:</b>	<b>Mental methods or strategies:</b> <b>Children should understand when to and be able to apply these strategies:</b>
<b>Year 1</b> <ul style="list-style-type: none"> <li>● doubles of all numbers to 10, e.g. double 6</li> <li>● odd and even numbers to 20</li> </ul>	<ul style="list-style-type: none"> <li>● count on from and back to zero in ones, twos, fives or tens</li> </ul>	<ul style="list-style-type: none"> <li>● use patterns of last digits, e.g. 0 and 5 when counting in fives</li> </ul>
<b>Year 2</b> <ul style="list-style-type: none"> <li>● doubles of all numbers to 20, e.g. double 13, and corresponding halves</li> <li>● doubles of multiples of 10 to 50, e.g. double 40, and corresponding halves</li> <li>● multiplication facts for the 2, 5 and 10 times-tables, and corresponding division facts</li> <li>● odd and even numbers to 100</li> </ul>	<ul style="list-style-type: none"> <li>● double any multiple of 5 up to 50, e.g. double 35</li> <li>● halve any multiple of 10 up to 100, e.g. halve 90</li> <li>● find half of even numbers to 40</li> <li>● find the total number of objects when they are organised into groups of 2, 5 or 10</li> </ul>	<ul style="list-style-type: none"> <li>● partition: double the tens and ones separately, then recombine</li> <li>● use knowledge that halving is the inverse of doubling and that doubling is equivalent to multiplying by two</li> <li>● use knowledge of multiplication facts from the 2, 5 and 10 times-tables, e.g. recognise that there are 15 objects altogether because there are three groups of five</li> </ul>

<p><b>Year 3</b></p> <ul style="list-style-type: none"> <li>● multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables, and corresponding division facts</li> <li>● doubles of multiples of 10 to 100, e.g. double 90, and corresponding halves</li> </ul>	<ul style="list-style-type: none"> <li>● double any multiple of 5 up to 100, e.g. double 35</li> <li>● halve any multiple of 10 up to 200, e.g. halve 170</li> <li>● multiply one-digit or two-digit numbers by 10 or 100, e.g. <math>7 \times 100</math>, <math>46 \times 10</math>, <math>54 \times 100</math></li> <li>● find unit fractions of numbers and quantities involving halves, thirds, quarters, fifths and tenths</li> </ul>	<ul style="list-style-type: none"> <li>● partition: when doubling, double the tens and ones separately, then recombine</li> <li>● partition: when halving, halve the tens and ones separately, then recombine</li> <li>● use knowledge that halving and doubling are inverse operations</li> <li>● recognise that finding a unit fraction is equivalent to dividing by the denominator and use knowledge of division facts</li> <li>● recognise that when multiplying by 10 or 100 the digits move one or two places to the left and zero is used as a place holder</li> </ul>
<p><b>Year 4</b></p> <ul style="list-style-type: none"> <li>● Multiplication facts to <math>10 \times 10</math> and the corresponding division facts</li> <li>● doubles of numbers 1 to 100, e.g. double 58, and corresponding halves</li> <li>● doubles of multiples of 10 and 100 and corresponding halves</li> <li>● fraction and decimal equivalents of one-half, quarters, tenths and hundredths, e.g. <math>\frac{3}{10}</math> is 0.3 and <math>\frac{3}{100}</math> is 0.03</li> <li>● factor pairs for known multiplication facts</li> </ul>	<ul style="list-style-type: none"> <li>● double any two-digit number, e.g. double 39</li> <li>● double any multiple of 10 or 100, e.g. double 340, double 800, and halve the corresponding multiples of 10 and 100</li> <li>● halve any even number to 200</li> <li>● find unit fractions and simple non-unit fractions of numbers and quantities, e.g. <math>\frac{38}{100}</math> of 24</li> <li>● multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. <math>325 \times 10</math>, <math>42 \times 100</math>, <math>120 \div 10</math>, <math>600 \div 100</math>, <math>850 \div 10</math></li> <li>● multiply a multiple of 10 to 100 by a single-digit number, e.g. <math>40 \times 3</math></li> <li>● multiply numbers to 20 by a single-digit, e.g. <math>17 \times 3</math></li> <li>● identify the remainder when dividing by 2, 5 or 10</li> </ul>	<ul style="list-style-type: none"> <li>● partition: double or halve the tens and ones separately, then recombine</li> <li>● use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right and zero is used as a place holder</li> <li>● use knowledge of multiplication facts and place value, e.g. <math>7 \times 8 = 56</math> to find <math>70 \times 8</math>, <math>7 \times 80</math></li> <li>● use partitioning and the distributive law to multiply, e.g. <math>13 \times 4 = (10 + 3) \times 4 = (10 \times 4) + (3 \times 4) = 40 + 12 = 52</math></li> </ul>

	<ul style="list-style-type: none"><li>● give the factor pair associated with a multiplication fact, e.g. identify that if <math>2 \times 3 = 6</math> then 6 has the factor pair 2 and 3</li></ul>	
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<p><b>Year 5</b></p> <ul style="list-style-type: none"> <li>● squares to <math>10 \times 10</math></li> <li>● division facts corresponding to tables up to <math>10 \times 10</math>, and the related unit fractions, e.g. <math>7 \times 9 = 63</math> so one-ninth of 63 is 7 and one-seventh of 63 is 9</li> <li>● percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths</li> <li>● factor pairs to 100</li> </ul>	<ul style="list-style-type: none"> <li>● multiply and divide two-digit numbers by 4 or 8, e.g. <math>26 \times 4</math>, <math>96 \div 8</math></li> <li>● multiply two-digit numbers by 5 or 20, e.g. <math>320 \times 5</math>, <math>14 \times 20</math></li> <li>● multiply by 25 or 50, e.g. <math>48 \times 25</math>, <math>32 \times 50</math></li> <li>● double three-digit multiples of 10 to 500, e.g. <math>380 \times 2</math>, and find the corresponding halves, e.g. <math>760 \div 2</math></li> <li>● find the remainder after dividing a two-digit number by a single-digit number, e.g. <math>27 \div 4 = 6</math> R 3</li> <li>● multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. <math>4.3 \times 10</math>, <math>0.75 \times 100</math>, <math>25 \div 10</math>, <math>673 \div 100</math>, <math>74 \div 100</math></li> <li>● multiply pairs of multiples of 10, e.g. <math>60 \times 30</math>, and a multiple of 100 by a single digit number, e.g. <math>900 \times 8</math></li> <li>● divide a multiple of 10 by a single-digit number (whole number answers) e.g. <math>80 \div 4</math>, <math>270 \div 3</math></li> <li>● find fractions of whole numbers or quantities, e.g. <math>\frac{23}{45}</math> of 27, <math>\frac{45}{70}</math> of 70 kg</li> <li>● find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80</li> <li>● find factor pairs for numbers to 100, e.g. 30 has the factor pairs <math>1 \times 30</math>, <math>2 \times 15</math>, <math>3 \times 10</math> and <math>5 \times 6</math></li> </ul>	<ul style="list-style-type: none"> <li>● multiply or divide by 4 or 8 by repeated doubling or halving</li> <li>● form an equivalent calculation, e.g. to multiply by 5, multiply by 10, then halve; to multiply by 20, double, then multiply by 10</li> <li>● use knowledge of doubles/halves and understanding of place value, e.g. when multiplying by 50 multiply by 100 and divide by 2</li> <li>● use knowledge of division facts, e.g. when carrying out a division to find a remainder</li> <li>● use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right relative to the decimal point, and zero is used as a place holder</li> <li>● use knowledge of multiplication and division facts and understanding of place value, e.g. when calculating with multiples of 10</li> <li>● use knowledge of equivalence between fractions and percentages, e.g. to find 50%, 25% and 10%</li> <li>● use knowledge of multiplication and division facts to find factor pairs</li> </ul>
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<p><b>Year 6</b></p> <ul style="list-style-type: none"> <li>● squares to <math>12 \times 12</math></li> <li>● squares of the corresponding multiples of 10</li> <li>● prime numbers less than 100</li> <li>● equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or 35/100</li> </ul>	<ul style="list-style-type: none"> <li>● multiply pairs of two-digit and single-digit numbers, e.g. <math>28 \times 3</math></li> <li>● divide a two-digit number by a single-digit number, e.g. <math>68 \div 4</math></li> <li>● divide by 25 or 50, e.g. <math>480 \div 25</math>, <math>3200 \div 50</math></li> <li>● double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2</li> <li>● multiply pairs of multiples of 10 and 100, e.g. <math>50 \times 30</math>, <math>600 \times 20</math></li> <li>● divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. <math>600 \div 20</math>, <math>800 \div 400</math>, <math>2100 \div 300</math></li> <li>● multiply and divide two-digit decimals such as <math>0.8 \times 7</math>, <math>4.8 \div 6</math></li> <li>● find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g</li> <li>● simplify fractions by cancelling</li> <li>● scale up and down using known facts, e.g. given that three oranges cost 24p, find the cost of four oranges</li> <li>● identify numbers with odd and even numbers of factors and no factor pairs other than 1 and themselves</li> </ul>	<ul style="list-style-type: none"> <li>● partition: use partitioning and the distributive law to divide tens and ones separately, e.g. <math>92 \div 4 = (80 + 12) \div 4 = 20 + 3 = 23</math></li> <li>● form an equivalent calculation, e.g. to divide by 25, divide by 100, then multiply by 4; to divide by 50, divide by 100, then double</li> <li>● use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division</li> <li>● recognise how to scale up or down using multiplication and division, e.g. if three oranges cost 24p: one orange costs <math>24 \div 3 = 8</math>p four oranges cost <math>8 \times 4 = 32</math>p</li> <li>● Use knowledge of multiplication and division facts to identify factor pairs and numbers with only two factors</li> </ul>
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